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Sensitivity of the DVCS Cross Section to the Compton form Factors in Scalar QED

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Abstract Using deeply virtual Compton scattering as a tool to study the structure of hadrons in an exclusive process, one expresses the amplitudes in terms of invariant quantities: the Compton form factors. In this paper the sensitivity of the hadronic part of the cross section to the Compton form factors is determined.

1 Introduction

Deeply-virtual Compton scattering (DVCS)[1,2] has been proposed to determine the generalized-parton distributions (GPDs) of hadrons.

A hard photon, $q^2 = -Q^2$, with Q much larger than the characteristic hadronic scales, probes the quark content of the hadronic target. The detection of the outgoing real photon provides information not contained in deep-inelastic scattering (DIS). It is important to realize that GPDs are not invariant quantities. They are related asymptotically, i.e., for large Q , and for small Mandelstam t to Compton form factors (CFFs) [3,4]. Even if the experimental data are analysed in terms of these Lorentz-invariant quantities, it is not immediately clear what are the sensitivities of the data to the CFFs. In a simple case, namely DVCS on a scalar target where the minimal number of diagrams that are necessary to maintain EM current conservation are known, the corrections to the tree-level case can be calculated. We have found their scaling with Q^2 . In practice, the DVCS amplitude interferes with the Bethe–Heitler amplitude. The latter one does only depend on the EM form factor(s) and thus give by itself no information about the CFFs. We shall concentrate on the DVCS part of the amplitude and even ignore the leptonic part, because it is independent of the CFFs.

2 Formalism

In virtual Compton scattering the physical amplitudes are written as contractions of a tensor with the polarization vectors of the photons:

$$A(h', h) = \epsilon^*(q'; h')_\mu T^{\mu\nu} \epsilon(q; h)_\nu \quad \text{with} \quad q'_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} q_\nu = 0 \quad (i.e., T^{\mu\nu} \text{ transverse}). \quad (1)$$

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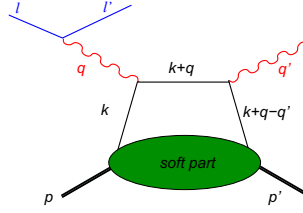


Fig. 1 Handbag diagram, including the leptonic part

The tensor is expressed in terms of CFFs and basis tensors. It is important to use the most general form of that tensor operator consistent with EM gauge invariance. We shall briefly discuss two proposals, one by Tarrach [1,2] and the other by Metz [3].

In the case we study, namely DVCS on a scalar hadron, the number of independent tensor structures is known, namely equal to five [1,2,5]. As these five independent tensor structures can be chosen in an infinite number of ways, we look for a synthetic way to construct the effective tensor.

Following Tarrach, one constructs the tensor $T^{\mu\nu}$ by applying a two-sided projector $\tilde{g}^{\mu\nu}(q, q')$ to the most general second rank tensor expressed in terms of our basis:

$$T^{\mu\nu} = \tilde{g}^{\mu m} t_{mn} \tilde{g}^{nv}, \quad t_{mn} = t_0 g_{mn} + \sum_{i,j} t_{ij} k_{im} k_{jn} \quad \text{with} \quad \tilde{g}^{\mu\nu}(q, q') = g^{\mu\nu} - \frac{q^\mu q'^\nu q \cdot q'}{q \cdot q'}. \quad (2)$$

We define the reduced momenta, $(k = \bar{P} = p + p', q', q)$:

$$\tilde{k}_L^\mu = \tilde{g}^{\mu m} k_m, \quad \tilde{k}_R^\nu = k_n \tilde{g}^{nv} \quad (3)$$

and find for unrestricted kinematics the following result for $T^{\mu\nu}$

$$T^{\mu\nu} = \mathcal{H}_0 \tilde{g}^{\mu\nu} + \mathcal{H}_1 \tilde{P}_L^\mu \tilde{P}_R^\nu + \mathcal{H}_2 \tilde{P}_L^\mu \tilde{q}_R^\nu + \mathcal{H}_3 \tilde{q}'_{\mu L} \tilde{P}_R^\nu + \mathcal{H}_4 \tilde{q}'_{\mu L} \tilde{q}_R^\nu. \quad (4)$$

Contracting the tensor with $\epsilon_\mu^*(q')$ and $\epsilon_\nu(q)$ we find that all five pieces of the tensor contribute, if $q'^2 \neq 0$ and $q^2 \neq 0$. The number of independent tensor structures is equal to the number of independent physical matrix elements consistent with parity conservation: $A(-h', -h) = (-1)^{h'-h} A(h', h)$, $h', h = \pm 1, 0$,

$$A(1, 1), A(1, 0), A(1, -1), A(0, 1), \text{ and } A(0, 0). \quad (5)$$

We limit ourselves here to the case where one of the photons is real, say $q'^2 = 0$. Then the number of independent physical amplitudes reduces to three, say $A(1, 1)$, $A(1, 0)$, and $A(1, -1)$. In this case the CFFs \mathcal{H}_3 and \mathcal{H}_4 do not contribute.

The method using the projectors introduces a kinematical singularity at $q' \cdot q = 0$. In Tarrach's paper a method is described to remove these kinematic poles. Here we give the final result of that algorithm as obtained in the thesis of Metz. His CFFs in the scalar case are denoted as $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$, and \mathcal{B}_{19} . They are implicitly given in terms of the elementary tensor by the following equations:

$$\begin{aligned} M^{\mu\nu} &= \mathcal{B}_1 M_1^{\mu\nu} + \mathcal{B}_2 M_2^{\mu\nu} + \mathcal{B}_3 M_3^{\mu\nu} + \mathcal{B}_4 M_4^{\mu\nu} + \mathcal{B}_{19} M_{19}^{\mu\nu}, \\ M_1^{\mu\nu} &= -q' \cdot q g^{\mu\nu} + q^\mu q^\nu, \\ M_2^{\mu\nu} &= -(\bar{P} \cdot q)^2 g^{\mu\nu} - q' \cdot q \bar{P}^\mu \bar{P}^\nu + \bar{P} \cdot q (\bar{P}^\mu q'^\nu + q^\mu \bar{P}^\nu), \\ M_3^{\mu\nu} &= q^2 q^2 g^{\mu\nu} + q' \cdot q q^\mu q^\nu - q^2 q^\mu q^\nu - q^2 q^\mu q^\nu, \\ M_4^{\mu\nu} &= \bar{P} \cdot q (q^2 + q^2) g^{\mu\nu} - \bar{P} \cdot q (q^\mu q^\nu + q^\mu q^\nu) \\ &\quad - q^2 \bar{P}^\mu q^\nu - q^2 q^\mu \bar{P}^\nu + q' \cdot q (\bar{P}^\mu q^\nu + q^\mu \bar{P}^\nu), \\ M_{19}^{\mu\nu} &= (\bar{P} \cdot q)^2 q^\mu q^\nu + q^2 q^2 \bar{P}^\mu \bar{P}^\nu - \bar{P} \cdot q q^2 q^\mu \bar{P}^\nu - \bar{P} \cdot q q^2 \bar{P}^\mu q^\nu. \end{aligned}$$

The relations between the CFFs \mathcal{H}_i and the CFFs \mathcal{B}_j is found by identifying $M^{\mu\nu}$ and $T^{\mu\nu}$. (We shall not need them here.) If $q'^2 = 0$, which is the case we treat here, the independent CFFs are $\mathcal{B}_{1,2}$ and \mathcal{B}_4 , which agrees with the number of independent amplitudes.

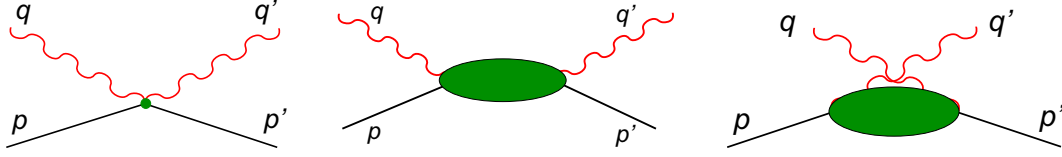


Fig. 2 Seagull, s - and u -channel amplitudes

The tree-level DVCS amplitude corresponding to Tarrach's basic formulation to the CFFs

$$\mathcal{H}_0^{\text{tree}} = -2, \quad \mathcal{H}_1^{\text{tree}} = \frac{1}{s - M^2} + \frac{1}{u - M^2}, \quad \mathcal{H}_3^{\text{tree}} = 0, \quad (6)$$

and in Metz's formulation to

$$\mathcal{B}_1^{\text{tree}} = \frac{1}{s - M^2} + \frac{1}{u - M^2}, \quad \mathcal{B}_2^{\text{tree}} = -\frac{2}{(s - M^2)(u - M^2)}, \quad \mathcal{B}_4^{\text{tree}} = 0, \quad (7)$$

$s = (p + q)^2$, $u = (p - q)^2$, M is the target hadron's mass.

3 Amplitudes

In a simple model where the internal structure of the target is included at one-loop level, it is known [6] that if all diagrams are included to guarantee EM gauge invariance to second order in the coupling constant g , one finds that the corrections at one-loop level scale as ($Q^2 = -q^2$)

$$\frac{1}{Q^n} \log(A + BQ^2), \quad (8)$$

where the functions A and B depend on the Mandelstam variables. The power n is 2 for \mathcal{H}_0 and 4 for $\mathcal{H}_{1,2}$. The imaginary parts scale as $1/Q^n$.

We shall not consider this model in the present work. Instead we use the tree-level CFFs in Metz's formulation. In this formulation the whole interval $0 \leq \theta \leq \pi$, where θ is the photon scattering angle, can be explored when the sensitivities of the cross sections to variations of the CFFs are studied. As we do not include the Bethe-Heitler amplitudes, we shall consider the VCS amplitudes only and consider variations by $\pm 10\%$ for \mathcal{B}_1 and \mathcal{B}_2 to study the sensitivity of the cross section to them. Furthermore, we add a third CFF, namely \mathcal{B}_4 for which we take 0 (the tree-level value) and $\pm 0.1 \mathcal{B}_2$.

We use the center-of mass (CMF) kinematics for the hadronic system and rotate the coordinate system such that the z -axis is along the three-momentum of the virtual photon.

To compute the amplitudes, we need the polarization vectors. We use circular polarisation in our calculation.

4 Numerical Calculations

We calculate the tree-level amplitudes for two realistic kinematical situations, described in a document written by Julie Roche and co-workers [7]. We take the kinematics for the largest value of the virtuality of the photon considered there, namely $Q^2 = 9 \text{ GeV}^2$, because DVCS is mostly used to learn about GPDs [1,2] and the connection of these quantities to the invariant CFFs is most accurate for large Q^2 . We chose a value $x_{\text{Bj}} = 0.6$ because this value lies in the centre of the kinematical domain considered in Ref. [7]. The smallest hadronic target with spin zero is the α -particle, thus we concentrate our study to the case where $M = 3.7272 \text{ GeV}/c^2$. Then we use the Metz tensor to calculate the amplitudes using the tree-level CFFs with variations of their numerical values in a range of $\pm 10\%$. The quantities we show in our results are *pseudo cross sections*, i.e., the sums of the squared amplitudes over the spin components.

We want to know in what kinematical region the greatest sensitivities of the *pseudo cross section*

$$\sum_{h'h} |A(h', h)|^2 \quad (9)$$

occurs. Therefore we need to explore for a given value of the Bjorken variable x_{Bj} and Q^2 the whole kinematical domain as parameterized by the photon scattering angle in the CMF $0 \leq \theta \leq \pi$. This means that $\theta = 0$ corresponds to the recoiled hadron moving in the backward direction in the CMF.

We stress that the parameterisation provided by Metz can be used in the whole kinematic range. The one given by Tarrach cannot, because there exists an angle where $q' \cdot q = 0$. Thus, the simplest construction will not be enough, unless one would limit oneself to a situation where $|t|$ is much smaller than Q^2 , which limits the domain in θ .

First, we show the three independent amplitudes at tree level: $A(1, 1)$, $A(1, 0)$, and $A(1, -1)$. It is clear that for these values of Q^2 and x_{Bj} the amplitude involving a longitudinal virtual photon is dominant.

It is interesting to note that the largest magnitude of the longitudinal amplitude $|A(1, 0)|$ occurs at the specific angle where the two transverse amplitudes coincide: $A(1, 1) = A(1, -1)$. This angle can be computed rather easily from the consideration of the maximum production of the real photon in the target-rest-frame (TRF). When the virtual photon with the longitudinal polarization impinges on the scalar target at rest, the real photon is most abundantly produced at right angles with respect to the incoming virtual photon. Using the Lorentz transformation from the TRF to the CMF we find that the cosine of the scattering angle is given by

$$\cos \theta = -\frac{Q\sqrt{Q^2 + 4x_{Bj}^2 M^2}}{Q^2 + 2Mx_{Bj}}. \quad (10)$$

From the values of Q^2 , M , and x_{Bj} given in Fig. 3 we can estimate that this angle is around 2.25, which is very close to $3\pi/4 \approx 2.36$. In the following figures we show the variations of the pseudo cross sections when the tree-level values of the CFFs are changed by $\pm 10\%$. Clearly, a variation of \mathcal{B}_1 with 10% has hardly visible effects on the cross section, as seen in Fig. 4.

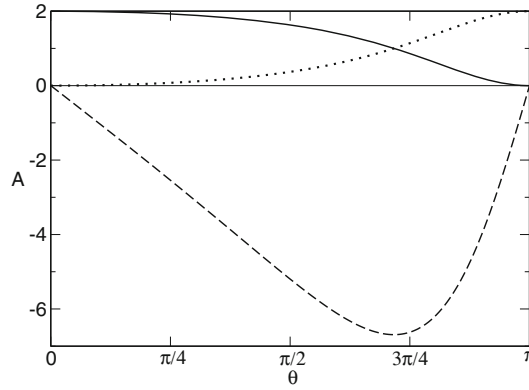


Fig. 3 The amplitudes $A(1, 1)$ (solid line), $A(1, 0)/i$ (dashed line), and $A(1, -1)$ (dotted line). $Q^2 = 9 \text{ (GeV/c)}^2$, $M = 3.7273 \text{ GeV/c}^2$, $x_{Bj} = 0.6$

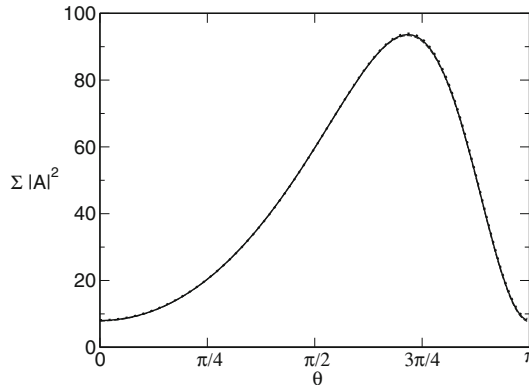


Fig. 4 Sum of squared amplitudes for $Q^2 = 9 \text{ (GeV/c)}^2$, $M = 3.7273 \text{ GeV/c}^2$, and $x_{Bj} = 0.6$. The CFF \mathcal{B}_1 is varied by $\pm 10\%$

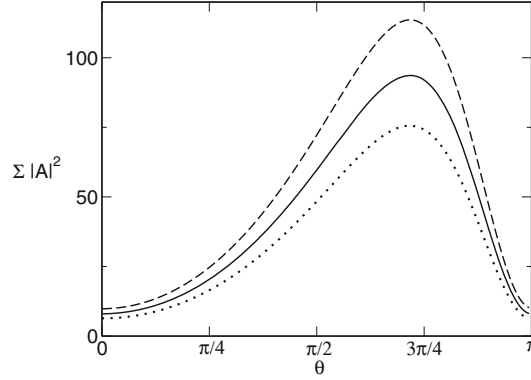


Fig. 5 Sum of squared amplitudes for $Q^2 = 9 \text{ (GeV}/c^2\text{)}^2$, $M = 3.7273 \text{ GeV}/c^2$, and $x_{\text{Bj}} = 0.6$. The CFF \mathcal{B}_2 is varied by $\pm 10\%$

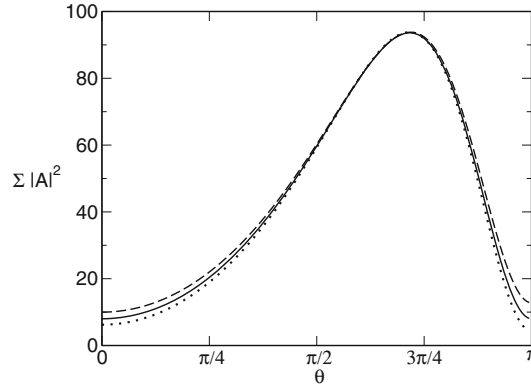


Fig. 6 Sum of squared amplitudes for $Q^2 = 9 \text{ (GeV}/c^2\text{)}^2$, $M = 3.7273 \text{ GeV}/c^2$, and $x_{\text{Bj}} = 0.6$. The CFF \mathcal{B}_4 is varied by $\pm 10\%$ of \mathcal{B}_2

On the contrary, Fig. 5 shows that the same relative variation of \mathcal{B}_2 has a dramatic effect on the cross section. Replacing the tree-level CFF \mathcal{B}_4 , which vanishes, to $\pm 0.1\mathcal{B}_2$, has only a significant effect on the cross section for extremely forward or backward angles, where the cross section itself is minimal.

This leads to the conclusion that the chance of determining all three CFFs is slim, even in this simplest possible case of a spinless target. The best one can do is to aim for the determination of \mathcal{B}_4 for angles in the extremely forward or backward kinematics.

Four more items should be considered before a final conclusion can be drawn. First, in the pseudo cross sections we have studied here, the kinematic factors connecting the differential cross section to the squares of the invariant amplitudes have not been included. Therefore, the kinematics where the cross section is most sensitive to the CFFs may differ somewhat from the ones indicated by our results.

Secondly, in an experiment, the complete amplitudes, hadronic plus Bethe–Heitler, must be used to calculate the cross section. Because the Bethe–Heitler amplitudes do not depend on the CFFs, but only on the well-measured EM form factor of the hadronic target, it would be surprising if the sensitivity of the cross section to the CFFs would be increased if the Bethe–Heitler amplitudes would be included.

Thirdly, in the full analysis, the interference term of the hadronic and the Bethe–Heitler amplitudes may be extracted for instance by measuring the beam-charge asymmetry, which may enhance the sensitivity to some of the CFFs in some kinematical domain.

Finally, because the cross sections are strongly varying with the scattering angle θ , one may expect that the CFFs may only be accurately determined from the data in a limited domain in θ , which corresponds for any fixed Bjorken variable x_{Bj} and Q^2 to a limited domain in the Mandelstam invariant t . Because the GPD description of DVCS implicitly assumes that Q^2 is much larger than any hadronic mass scale and that $|t|$ is much smaller than Q^2 , the limitations on the domain where the CFFs can be reliably determined may not have sufficient overlap with the domain where the GPD description makes sense.

5 Summary and Conclusions

- In the comparatively simple case of DVCS on a scalar target, it is not clear that one can disentangle the differential cross section to determine all three CFFs.
- Only the interference with the Bethe–Heitler may give additional information on the CFFs.
- The amplitudes $A(\pm 1, 0)$ are dominant. In the hemisphere where the recoiled target moves forward, these amplitudes are most sensitive to \mathcal{B}_2 .
- Using a non-singular basis like Metz’s is essential to the analysis of the differential cross section data in the whole kinematical domain.

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